ST. STEPHEN'S GIRLS' COLLEGE
Final Examination 2019 – 2020

FORM 4
143 students **LHK, KAL, CYN, YLN**

MATHEMATICS
Time allowed: 2 hours
Question/Answer Paper

Please read the following <u>instructions</u> very carefully.

1. Write your class, class number and name in the spaces provided on this cover.

2. This paper consists of TWO sections, A and B. Section A carries 36 marks and Section B carries 64 marks. **Attempt ALL questions in this paper.**

3. For **Section A**, you should put your answers on the **"Multiple Choice Answer Sheet"** provided. Note that you may only mark **ONE** answer for each question. Two or more answers will score **NO MARKS**.

4. For **Section B**, write your answers in the spaces provided in this **Question/Answer Paper.**

5. Graph paper and supplementary answer sheets will be supplied on request. Write your class, class number and name on each sheet, and they should be stapled to this paper.

6. Unless otherwise specified, all working must be clearly shown.

7. Unless otherwise specified, numerical answers should either be exact or correct to 3 significant figures.

8. The diagrams in this paper are not necessarily drawn to scale.

Class	
Class No.	
Name	
Division	

	Marker's Use Only	
A		
B	25	
	26	
	27	
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	37	
	38	
Total		/ 100

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SECTION A (36 marks, all questions carry equal marks): You are advised to spend 40 minutes on this section.

1. $\left(27\cdot 9^{n-1}\right)^3 =$

A. 3^{6n+3}. B. 3^{6n+15}.
C. 3^{9n+3}. D. 3^{9n+18}.

2. $al-bl+am-bm-an+bn =$

A. $(a+b)(l+m-n)$. B. $(a+b)(l-m+n)$.
C. $(a-b)(l+m-n)$. D. $(a-b)(l-m+n)$.

3. If the quadratic equation $k=\dfrac{x^2+7x+12}{3}$ has no real roots, find the range of values of k.

A. $k<-\dfrac{1}{12}$ B. $k<12$
C. $k>\dfrac{1}{12}$ D. $k>12$

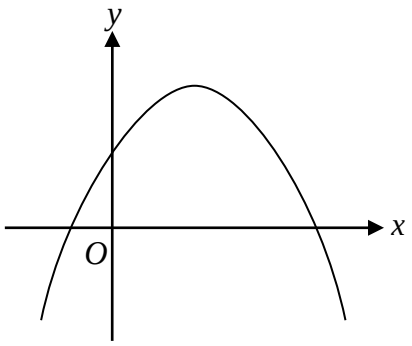
4. If b is a positive constant, then the root(s) of the equation $(x-3)^2=b^2$ is/are
A. $3-b$. B. $3+b$.
C. $-3+b$ or $-3-b$. D. $3+b$ or $3-b$.

5. If α and β are roots of the equation $x^2-3x+5=0$, find $\dfrac{1}{\alpha^2}+\dfrac{1}{\beta^2}$.

A. $-\dfrac{9}{25}$ B. $-\dfrac{1}{25}$
C. $\dfrac{1}{25}$ D. $\dfrac{9}{25}$

6. The figure shows a graph of $y=ax^2+bx+c$, where a, b and c are real constants. Which of the following is/are correct?

I. $b^2-4ac<0$
II. $b>0$
III. $c>0$



A. I only
B. I and II only
C. II and III only
D. I, II and III

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7. If α is a root of the quadratic equation $x^2+3x-5=0$, then $2\alpha^2+6\alpha+7=$

A. -3. B. 2.
C. 12. D. 17.

8. The cost of making x model cars is $\$(x^2+14x+116)$. Given the budget of \$356, how many model cars can be made?

A. 10 B. 16
C. 24 D. 34

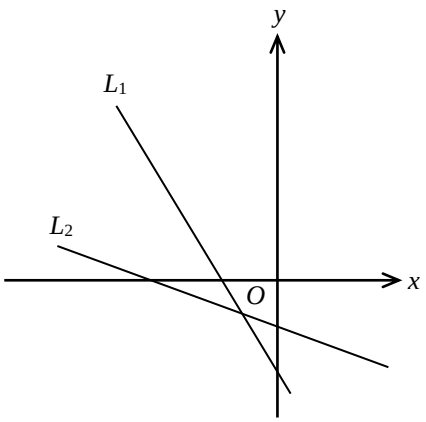
9. It is given that $f(x)=x^2+x+3$. $f(2a-1)=$

A. $4a^2+2a+2$. B. $4a^2-2a+3$.
C. $2a^2+2a+2$. D. $2a^2+2a$.

10. In the figure, the equations of the straight lines L_1 and L_2 are $x + ay = b$ and $bx + cy = d$ respectively. Which of the following is/are true?

I. $b < 0$
II. $ab < c$
III. $bc > ad$

A. I and II only
B. I and III only
C. II and III only
D. I, II and III



11. Let p be a constant such that $2x^4+px^3-4x-16$ is divisible by $2x+p$. Find p.

A. -2 B. 2
C. 4 D. 8

12. Let $f(x)=x^9-2x+r$, where r is a constant. If $f(x)$ is divisible by $x+1$, find the remainder when $f(x)$ is divided by $x-1$.

A. 0 B. -1
C. -2 D. 2

13. Let $f(x)=x^3+5x^2-2x-24$. If $f(-3)=f(2)=0$, factorize $f(x)$.

A. $(x+2)(x-3)(x-4)$ B. $(x-2)(x-3)(x+4)$
C. $(x+2)(x+3)(x-4)$ D. $(x-2)(x+3)(x+4)$

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14. If the L.C.M. of $x^3y^6z^2$ and another polynomial is $2x^4y^8z^2$, which of the following may be this polynomial?

I. $2x^4y^6z$

II. $2x^4y^8z$

III. $2x^4y^8z^2$

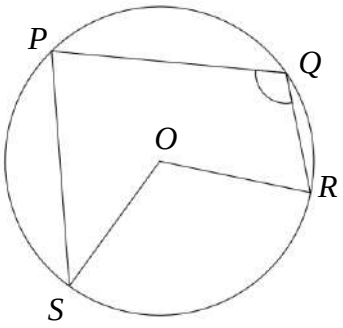
A. II only
B. III only
C. I and II only
D. II and III only

15. If $8^a = 32^b$, then $a:b =$

A. $4:1$.
B. $1:4$.
C. $5:3$.
D. $3:5$.

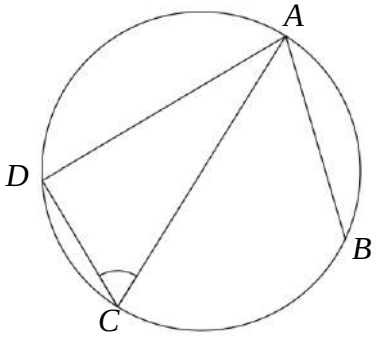
16. In the figure, O is the centre of the circle $PQRS$. If $\angle ROS = 114°$, $\angle QPS = 80°$ and $PQ = PS$, find $\angle PQR$.

A. 100°
B. 107°
C. 112°
D. 137°



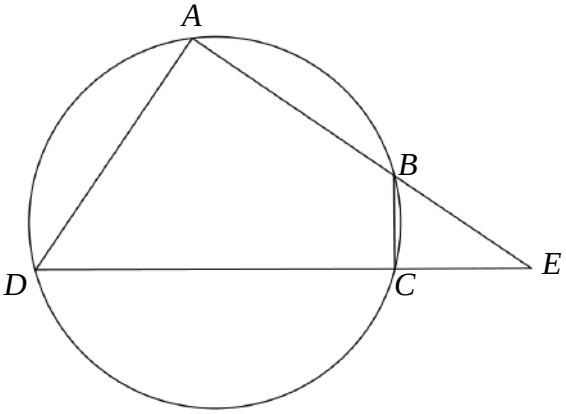
17. In the figure, $ABCD$ is a circle. If AC is a diameter of the circle, $\angle BAC = 45°$ and $\overset{\frown}{DC} : \overset{\frown}{CB} = 2:3$, find $\angle ACD$.

A. 30°
B. 45°
C. 60°
D. 75°



18. In the figure, $ABCD$ is a circle. AB produced and DC produced meet at E and $BC \perp DE$. If BC = 12 cm, AD = 36 cm and BE = 20 cm, find AB.

A. 28 cm
B. 30 cm
C. 32 cm
D. 34 cm



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19. If the straight lines $hx + 6y + 18 = 0$ and $2x + ky + 12 = 0$ are perpendicular to each other and intersect at a point on the y-axis, then $h =$

A. -12.
B. -3.
C. 3.
D. 12.

20. The maximum value of $\dfrac{45}{2\sin^2 x + 3} =$

A. 45.
B. 15.
C. 9.
D. 5.

21. Find the value of $\sin^2 1° + \sin^2 2° + \sin^2 3° + ... + \sin^2 90 + \sin^2 91 + ... + \sin^2 179° + \sin^2 180°$.

A. 90
B. 45.5
C. 45
D. 22.5

22. Simplify $\dfrac{\sin(270° + \theta)}{\tan(180° - \theta)} \times \sin(360° - \theta)$.

A. $\sin^2 \theta$
B. $-\cos^2 \theta$
C. $\tan \theta$
D. $-\tan \theta$

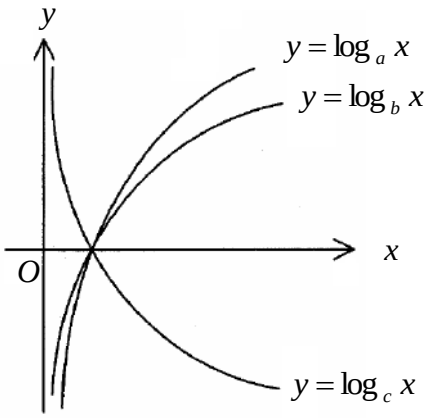
23. Which of the following is the best estimate of 2020^{2019}?

A. 10^{8076}
B. 10^{8000}
C. 10^{6674}
D. 10^{4039}

24. The figure shows the graphs of $y = \log_a x$, $y = \log_b x$ and $y = \log_c x$ on the same rectangular coordinate system, where a, b and c are positive constants. It is given that the graph of $y = \log_b x$ and the graph of $y = \log_c x$ are reflection images of each other in the $x-$axis. Which of the following is/are correct?

I. $a > 1$
II. $c = \dfrac{1}{b}$
III. $b > a$

A. I and II only
B. I and III only
C. II and III only
D. I, II and III



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SECTION B (64 marks)

25. Factorize (3 marks)
    (a) $x^2 + 3x - 4$,
    (b) $x^4 + 3x^3 - 4x^2$.

26. Make b the subject of the formula $\frac{2a}{b} + \frac{a}{5} = \frac{3}{b}$. (3 marks)

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27. Simplify $\frac{x}{2x-1}+\frac{x^2+4x-3}{2x^2+x-1}$. (5 marks)

28. Solve the exponential equation $\frac{2^{2x}\bullet 4^{x+1}}{8^{x+2}}=16$. (4 marks)

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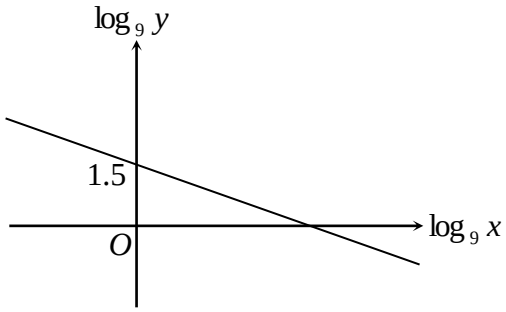
29. Solve the equation $5^{x+2} = 8^x$. (3 marks)

30. The graph in the figure shows the linear relation between $\log_9 x$ and $\log_9 y$. The slope and the intercept on the vertical axis of the graph are -0.5 and 1.5 respectively. Express the relation between x and y in the form of $y = Ax^k$, where A and k are constants. (4 marks)



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31. It is given that $\frac{1}{\alpha} = 1 + ki$, where k is a real number. If the imaginary part of α is $-\frac{2}{5}$, find the possible values of k.

(3 marks)

32. Solve $3\sin^2 x + \cos x \sin x - 2\cos^2 x = 0$ for $0° \le x \le 360°$. (4 marks)

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33. $x+3$ is a factor of the cubic polynomial $f(x)$. When $f(x)$ is divided by x^2-9, the remainder is $px+12$, where p is a constant.

(a) Find p. (3 marks)

(b) $f(x)$ is divisible by $x-4$. When $f(x)$ is divided by x, the remainder is 12.

Are all the roots of the equation $f(x)=0$ integers? Explain your answer. (4 marks)

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34. If α and β are the roots of the quadratic equation $3x(x+3)=8$, form a quadratic equation in x with integral coefficients whose roots are $4-\alpha$ and $4-\beta$. (5 marks)

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35. The total area of two squares is 225 cm^2 and the difference of their perimeters is 12 cm.
    (a) If the side of the larger square is x cm each, express the length of each side of the smaller square in terms of x. (2 marks)
    (b) Hence, find the length of the side of the larger square. (3 marks)

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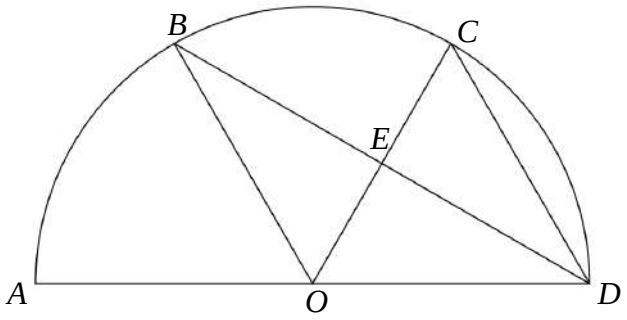
36. In the figure, O is the centre of the semicircle $ABCD$. OC is the angle bisector of $\angle BOD$ and OC cuts BD at E.



(a) Prove that $\Delta OBE \cong \Delta ODE$. (2 marks)

(b) If OB // DC, find $\angle OBD$. (3 marks)

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37. The straight line L_1 passes through $A(4, 3)$ and $B(8, 0)$. Another straight line L_2 is perpendicular to L_1 and cuts the y-axis at $(0, 16)$. L_1 and L_2 intersect at P.
    (a) Find the equation of L_2. (2 marks)
    (b) Find the coordinates of P. (3 marks)
    (c) It is given that L_2 cuts the x-axis at C. Find the coordinates of the circumcentre of ΔPBC. (2 marks)

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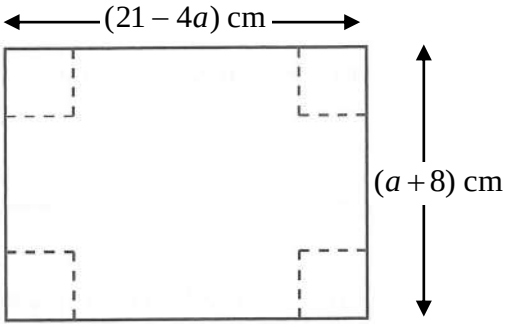
38. (a) Let $g(x) = \frac{13}{4}x - x^2$. Using the method of completing the square, find the coordinates of the vertex of the graph of $y = g(x)$. (2 marks)

(b) In the figure, a small square of side 4 cm is cut from each corner of a rectangular card of dimension $(a+8)\text{ cm} \times (21-4a)\text{ cm}$. The remaining part of the card is then folded to form an open box of capacity $V\text{ cm}^3$.



(i) Express V in terms of a.

(ii) Wallace claims that the capacity of the box must be smaller than 45 cm^3. Do you agree? Explain your answer.

(4 marks)

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***** END OF PAPER *****

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