Form 4   **YLN**

Mathematics Extended Part Module 1 (Calculus and Statistics)
Time allowed: 45 minutes
Total marks: 35
Question/Answer Paper

Please read the following <u>instructions</u> very carefully.

1. Write your class, class number and name in the spaces provided on this cover.

2. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question/Answer Paper.

3. Unless otherwise specified, all working must be clearly shown.

4. Unless otherwise specified, numerical answers should be exact or given to **4 decimal places**.

Class	
Class No.	
Name	

	Marker's Use
1	
2	
3	
4	
5	
6	
Total	/35

1. (a) Expand $(1+e^{-5x})^3$ in ascending powers of x as far as the term in x^2.   (4 marks)

    (b) Find the coefficient of x^2 in the expansion of $(x+3)^5(1+e^{-5x})^3$.   (2 marks)

2. (a) Expand e^{-12x} in ascending powers of x as far as the term in x^2. (1 marks)

(b) Let n be a positive integer. If the coefficient of x^2 in the expansion of

$e^{-12x}(1+4x)^n$ is -8, find the values of n. (4 marks)

3. Evaluate the following limits.

(a) $\displaystyle\lim_{x\to 3}\frac{x-3}{\sqrt{x}-\sqrt{3}}$ (2 marks)

(b) $\displaystyle\lim_{x\to+\infty}\frac{x^3e^{-2x}+4x^4e^{-x}}{x^4e^{-x}+2x^3e^{-4x}}$ (2 marks)

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4. Let $y = x^2 e^{-x^2+1}$.

   (a) Find y' and y'' .   (4 marks)

   (b) Simplify $y'' + (2x-1)\,y' + (-2x+6)y$.   (1 mark)

5. Consider the curve $\quad C: y = \dfrac{x+5}{\sqrt{x-3}}$, where $x > 3$.

   (a)   Find $\dfrac{dy}{dx}$.   (2 marks)

   (b)   A tangent to C passes through the point $(31, 0)$.   Find the slope of the tangent.

                                                                                          (5 marks)

6. Wallace studies the number of bacteria P (in ten thousand) in an experiment by

$$P = \frac{4500}{ae^{bt} + 5}$$

where a and b are constants and t ($t \geq 0$) is the number of months elapsed since the start of the study.

(a) Express $\ln\left(\dfrac{4500}{P} - 5\right)$ as a linear function of t. (2 marks)

(b) Wallace finds that the intercept on the horizontal axis and that on the vertical axis of the graph of the linear function obtained in (a) are $100 \ln 4$ and $\ln 4$ respectively.

    (i) Find a and b.

    (ii) Find $\dfrac{dP}{dt}$ and $\dfrac{d^2 P}{dt^2}$.

(6 marks)