ST. STEPHEN'S GIRLS' COLLEGE
Mid-Year Examination 2021 – 2022

Form 4 **LC**

18 students

Mathematics Extended Part Module 1 (Calculus and Statistics)
Time allowed: 45 minutes
Total marks: 35
Question/Answer Paper

Please read the following <u>instructions</u> very carefully.

1. Write your class, class number and name in the spaces provided on this cover.

2. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question/Answer Paper.

3. Unless otherwise specified, all working must be clearly shown.

4. Unless otherwise specified, numerical answers should be exact or given to **4 decimal places**.

Class	
Class No.	
Name	

	Marker's Use
1	/7
2	/7
3	/6
4	/6
5	/9
Total	/35

1. Let n be a positive integer.

   (a) Expand $(x+3x^2)^n$ in ascending powers of x as far as the term in x^{n+2}.

   (b) It is given that the coefficient of x^{n+2} in the expansion of $(x+3x^2)^n$ is 90. Find the value of n.

   (c) Find the coefficient of x^7 in the expansion of $\dfrac{(x+3x^2)^n}{e^{2x}}$. (7 marks)

2. Let a and k be real constants.

(a) Express $f(x) = (1 + ae^{-kx})^3$ in ascending powers of x up to the term in x^2.

(b) If the constant term in the expansion of $f(x)$ is 8 , find the value of a.

(c) If the coefficient of x in the expansion of $f(x)$ is -12, find the coefficient of x^2.

(7 marks)

3. (a) Show that $3^{\frac{-2x}{\ln 3}} = e^{-2x}$ for all values of x.

(b) Hence, otherwise, solve $\begin{cases} y = 2 \cdot 3^{\frac{-2x}{\ln 3}} \\ y = 7e^{-x} + 15 \end{cases}$. (6 marks)

4. Let $h(x) = \log_5 x^4 + \dfrac{(\ln x)^2 - \log_2 32}{\ln 5}$, where $x > 0$. It is given that $h(x) = 0$ has two real roots α and β, where $\alpha < \beta$. Find the values of α and β in terms of e. (6 marks)

5. A researcher studies the growth of a plant on the surface of a pond. The area A (in thousand m^2) of the pond covered by the plant can be modelled by

$$A = \frac{45a^{kt-2}}{2a^{kt-2}+1},$$

where t ($t \geq 0$) is the number of months elapsed since the start of the study, and a and k are constants.

(a) Express $\ln\left(\frac{45}{A}-2\right)$ as a linear function of t. (2 marks)

(b) It is known that the slope and the intercept on the horizontal axis of the linear function in (a) are $-\ln\sqrt[5]{3}$ and 10 respectively.

(i) Find the values of a and k.

(ii) Find the area of the pond covered by the plant initially.

(iii) It is given that A is an increasing function. The researcher claims that the area of the pond covered by the plant cannot exceed 23000 m^2 in the long run. Do you agree? Explain your answer.

(7 marks)

End of Paper