## ST. STEPHEN'S GIRLS' COLLEGE
## Mid-Year Examination 2019 – 2020

**FORM 4**
**154 students**                                                **LHK, KAL, CYN, YLN**

## MATHEMATICS
### Time allowed : 1 hour 30 minutes
### Question/Answer Paper

**Please read the following <u>instructions</u> very carefully.**

| Class | |
|---|---|
| **Class No.** | |
| **Name** | |

1. Write your class, class number and name in the spaces provided on this cover.

2. This paper consists of TWO sections, A and B. Section A carries 24 marks and Section B carries 56 marks. **Attempt ALL questions in this paper.**

3. For **Section A**, you should put your answers on the **"Multiple Choice Answer Sheet"** provided. Note that you may only mark **ONE** answer for each question. Two or more answers will score **NO MARKS**.

4. For **Section B**, write your answers in the spaces provided in this **Question/Answer Paper.**

5. Graph paper and supplementary answer sheets will be supplied on request. Write your class, class number and name on each sheet, and they should be stapled to this paper.

6. Unless otherwise specified, all working must be clearly shown.

7. Unless otherwise specified, numerical answers should either be exact or correct to 3 significant figures.

8. The diagrams in this paper are not necessarily drawn to scale.

| | **Marker's Use Only** | |
|---|---|---|
| **A** | | |
| | 13 | |
| | 14 | |
| | 15 | |
| | 16 | |
| | 17 | |
| | 18 | |
| **B** | 19 | |
| | 20 | |
| | 21 | |
| | 22 | |
| | 23 | |
| | 24 | |
| | 25 | |
| **Total** | | **/ 80** |

**SECTION A (24 marks, all questions carry equal marks): You are advised to spend 20 minutes on this section.**

1.    The straight line $3x + ky + 1 = 0$ passes through $(2, -1)$. Find the slope of the line.

    A.  3                               B.  $-7$

    C.  $\dfrac{3}{7}$                      D.  $-\dfrac{3}{7}$

2.    The coordinates of the points $A$ and $B$ are $(9, -2)$ and $(-1, 8)$ respectively.  If $C$ is a point lying on the straight line $x - 2y = 0$ such that $AC = BC$, then the $x$-coordinate of $C$ is

    A.  1.                              B.  2.

    C.  3.                              D.  4.

3.    Solve $(6x-1)(2x+7) = (4-5x)(1-6x)$.

    A.  $x = \dfrac{1}{2}$ or $x = 3$          B.  $x = -\dfrac{1}{6}$ or $x = \dfrac{11}{3}$

    C.  $x = -\dfrac{1}{6}$ or $x = 3$        D.  $x = \dfrac{1}{6}$ or $x = \dfrac{11}{3}$

4.    It is given that $p$ is a constant. Solve the quadratic equation $4x^2 - 2x + \dfrac{p-1}{4} = 0$.

    A.  $x = \dfrac{-1 \pm \sqrt{2-p}}{4}$        B.  $x = \dfrac{-1 \pm \sqrt{2+p}}{2}$

    C.  $x = \dfrac{1 \pm \sqrt{2-p}}{4}$         D.  $x = \dfrac{1 \pm \sqrt{2+p}}{4}$

5.    It is given that $-5$ is a root of the quadratic equation $(x+k)^2 + x - 4 = 0$, where $k$ is a constant. Find all the possible values of $k$.

    A.  0 or 2                          B.  0 or 6

    C.  2 or 6                          D.  2 or 8

6.    It is given that $k$ is a real number.  The real part of $\dfrac{3-4k}{1-i} + \dfrac{k+i}{2i} =$

    A.  $2-2k$ .                        B.  $1-2k$ .

    C.  $\dfrac{3-5k}{2}$ .                 D.  $\dfrac{3-4k}{2}$ .

7.  Let $p$ be a real constant.  If the quadratic equation $5x^2 + 4px - 2p = 0$ has equal real roots, then $p =$

    A.  $-2.5$ .                          B.  $2.5$ .

    C.  $-2.5$ or $0$ .                    D.  $0$ or $2.5$ .

8.  Let $f(3x-2) = 4x^2 + 2x$.   $f(7) =$

    A.  210.                              B.  42.

    C.  $\dfrac{130}{9}$ .                 D.  19.

9.  The figure shows the graph of a quadratic function $y = ax^2 + bx + c$, where $a$ , $b$ and $c$ are constants.  Which of the following is/are correct?

        I.      $a < 0$
        II.     $b > 0$
        III.    $c > 0$

    A.  I and II only                     B.  II and III only

    C.  I and III only                    D.  I, II and III

10. For $0° \le x \le 360°$ , how many roots does the equation $2\cos^2 x = 1 - \sin x$  have?

    A.  2                                 B.  3

    C.  4                                 D.  5

11. Which of the following must be true?
        I.      $\cos(90° - \theta)\sin\theta = 1 - \sin^2(90° - \theta)$
        II.     $\tan\theta \tan(90° - \theta) = -1$
        III.    The maximum value of $\tan\theta$ is 1 for $0° \le \theta < 90°$ .

    A.  I only                            B.  II only

    C.  I and III only                    D.  II and III only

12. In the figure, two straight lines $ax + by = c$ and $dx + ey = f$ intersect at a point lying on the $y$-axis, where $c$ and $f$ are negative constants.  Which of the following must be true?

        I.      $bf = ce$
        II.     $cd > af$
        III.    $ae < bd$

    A.  I and II only                     B.  I and III only

    C.  II and III only                   D.  I, II and III

**SECTION B (56 marks)**

13.  Make $x$ the subject of the formula $\dfrac{a}{x} - \dfrac{b}{y} = 2$ .                 (3 marks)

14.  Simplify $\dfrac{\left(p^3 q^{-1}\right)^4}{\left(p^{-2}\right)^5}$ and express your answer with positive indices.                 (3 marks)

15.  Factorize
     (a)  $x^2 - 2xy - 3y^2$ ,
     (b)  $x^2 - 2xy - 3y^2 + 5x - 15y$ .                 (3 marks)

16. The equation of the straight line $L$ is $2x + 3y - 5 = 0$. Find the equation of the straight line that is parallel to $L$ and passes through $(4, -2)$.                    (3 marks)

17. The difference between two negative numbers is 2 and their product is 63. Find the two numbers.                    (3 marks)

18.  (a)  Solve $10x^2 + 9x - 22 = 0$ .                                   (2 marks)

     (b)  Tom deposited $10 000 in a bank at the beginning of 2017 and $9 000 at the beginning of 2018. The interest is compounded yearly at $r$% p.a., and the total amount he received at the beginning of 2019 is $22 000. Find $r$.                   (4 marks)

19.  It is given a quadratic equation $x^2 - (5m+2)x + (5m-1) = 0$, where $m$ is a real constant.

     (a)  Express the discriminant of the equation in terms of $m$.       (2 marks)

     (b)  Wallace claims that the equation may have two equal real roots for some real values of $m$.  Is the claim correct?  Explain your answer.         (2 marks)

20. If $\alpha$ and $\beta$ are the roots of the equation $x^2 + 8x + 35 = 0$, find the value of the expression
    $\dfrac{\alpha+1}{\beta} + \dfrac{\beta+1}{\alpha}$.                                           (4 marks)

21. Simplify $\dfrac{\sin 150°}{1 + \sin(270° + \theta)} + \dfrac{\cos 120°}{1 + \cos(360° - \theta)}$.                     (4 marks)

22.   Let $f(x) = x^2 + kx - 5$, where $k$ is a real constant.  It is given that $f(k) = f(k+12)$.

   (a)   Find the value of $k$.                                                      (3 marks)

   (b)   Let $g(x) = \dfrac{1}{f(x)}$.  Mr. Ng claims that domain of $g(x)$ can be all real numbers.

         Is the claim correct?  Explain your answer.                                 (2 marks)

22.   Let $f(x) = x^2 + kx - 5$, where $k$ is a real constant.  It is given that $f(k) = f(k+12)$.

23.  The SSGC annual school concert in 2019 will be held in the City Stadium.  The City Stadium can accommodate at most 15 000 people.  If the price of each ticket is $300, all 15 000 tickets will be sold.  For every increase of $1 in the ticket price, the number of tickets sold will decrease by 30.  Let $p$ be the price of each ticket, where $p \geq 300$ and the total income of the concert be $T$.

(a)  Show that $T = -30p^2 + 24000\,p$.                                     (2 marks)

(b)  Using the method of completing the square, find the price of the ticket in order to maximize the total income of the annual concert.                    (3 marks)

24. The straight lines $L_1: kx - y + 10 = 0$ and $L_2: x + 3y + 20 = 0$ are perpendicular to each other, and they intersect at point $A$.
    (a) Find the value of $k$. (2 marks)
    (b) Find the coordinates of $A$. (2 marks)
    (c) Suppose $L_1$ cuts the $y$-axis at $B$ and $L_2$ cuts the $x$-axis at $C$. Find the equation of the median of $\triangle ABC$ which passes through $A$. (3 marks)

25. It is given that $\tan\theta = \dfrac{1}{k}$, where $180° < \theta < 270°$ and $k \neq 0$.

    (a) Express $\dfrac{\cos\theta - \sin\theta}{2\sin\theta + \cos\theta}$ in terms of $k$. (3 marks)

    (b) If $\dfrac{\cos\theta - \sin\theta}{2\sin\theta + \cos\theta} = k - 1$, find $\theta$. (3 marks)

**\*\*\*\*\* END OF PAPER \*\*\*\*\***