ST. STEPHEN'S GIRLS' COLLEGE
Mid-Year Examination 2018 – 2019

FORM 4
149 students **LC, LHK, SCHL**

MATHEMATICS
Time allowed : 1 hour 30 minutes
Question/Answer Paper

Please read the following <u>instructions</u> very carefully.

1. Write your class, class number and name in the spaces provided on this cover.

2. This paper consists of TWO sections, A and B. Section A carries 24 marks and Section B carries 56 marks. **Attempt ALL questions in this paper.**

3. For **Section A**, you should put your answers on the **"Multiple Choice Answer Sheet"** provided. Note that you may only mark **ONE** answer for each question. Two or more answers will score **NO MARKS**.

4. For **Section B**, write your answers in the spaces provided in this **Question/Answer Paper.**

5. Graph paper and supplementary answer sheets will be supplied on request. Write your class, class number and name on each sheet, and they should be stapled to this paper.

6. Unless otherwise specified, all working must be clearly shown.

7. Unless otherwise specified, numerical answers should either be exact or correct to 3 significant figures.

8. The diagrams in this paper are not necessarily drawn to scale.

Class	
Class No.	
Name	

	Marker's Use Only	
A		
B	13	
	14	
	15	
	16	
	17	
	18	
	19	
	20	
	21	
	22	
	23	
Total		/ 80

SECTION A (24 marks, all questions carry equal marks): You are advised to spend 20 minutes on this section.

1. If $\frac{a}{1+y}=\frac{b}{1-y}$, then $y=$

   A. $\frac{a-b}{a+b}$.
   B. $\frac{b-a}{a+b}$.
   C. $\frac{a+b}{a-b}$.
   D. $\frac{a+b}{b-a}$.

2. Let k be a constant. Find the range of values of k such that the quadratic equation $x^2+4x+k=5$ has no real roots.

   A. $k>1$
   B. $k<1$
   C. $k>9$
   D. $k<9$

3. Let k be a constant. Solve the equation $(x-k)=(x+k-1)(x-k)$

   A. $2-k$
   B. $1-k$
   C. k or $2-k$
   D. k or $1-k$

4. Which of the following statements about the graph of $y=16-(x-6)^2$ is true?

   A. The equation of the axis of symmetry of the graph is $x=-6$.
   B. The y-intercept of the graph is 16.
   C. The graph opens upwards.
   D. The coordinates of the vertex of the graph is (6 , 16).

5. If $\frac{2x-3}{x+2}=\frac{3x-2}{x-4}$, then $x=$

   A. 1 or −16.
   B. −1 or 16.
   C. 1 or 16.
   D. −1 or −16.

6. The equation of the straight line L_1 is $4x-5y-40=0$. The straight line L_2 is perpendicular to L_1 and intersects L_1 at a point lying on the x-axis. Find the area of the region bounded by L_1, L_2 and the y-axis.

   A. 90
   B. 102.5
   C. 150.5
   D. 180

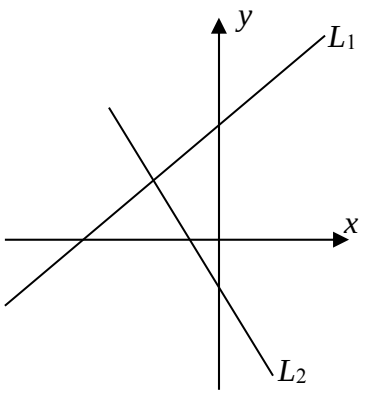
7. The coordinates of the points P and Q are (2, 4) and (8, −2) respectively. If R is a point lying on the straight line $x-3y=0$ such that $PR=QR$, then the x-coordinates of R is

   A. −3 .
   B. 2 .
   C. 5 .
   D. 6 .

8. In the figure, the equations of the straight lines L_1 and L_2 are $ax+2y=b$ and $x+cy=d$ respectively. Which of the following is/are true?

   I. $c<0$ II. $ad<b$ III. $ac>2$

   A. I and II only
   B. I and III only
   C. II only
   D. II and III only.



9. For $0° \le \theta < 360°$, how many distinct roots does the equation $\sin\theta + \tan\theta = 0$ have?
   A. 1 B. 2
   C. 3 D. 4

10. If $\cos\theta = -k$, where $180° \le \theta < 270°$, $\tan\theta =$
   A. $\frac{k}{\sqrt{1+k^2}}$. B. $\sqrt{\frac{1-k^2}{1+k^2}}$.
   C. $\frac{\sqrt{1+k^2}}{k}$. D. $\frac{\sqrt{1-k^2}}{k}$.

11. If the roots of the quadratic equation $x^2 - kx + 2 = 0$ are α and β, then $\alpha^3 + \beta^3 =$
   A. k^3. B. $k^3 - 2k$.
   C. $k^3 - 4k$. D. $k^3 - 6k$.

12. If k and $\frac{5}{3-i} + ki$ are real numbers, then $k =$
   A. $-\frac{1}{2}$. B. −2.
   C. $\frac{1}{2}$. D. 2.

SECTION B (56 marks)

13. Factorize
   (a) $a^3 + a^2b - 5a^2$,
   (b) $a^3 + a^2b - 5a^2 - a - b + 5$. (3 marks)

14. Make m the subject of the formula $\frac{m-3}{4}-\frac{n+5}{2}=0$. (3 marks)

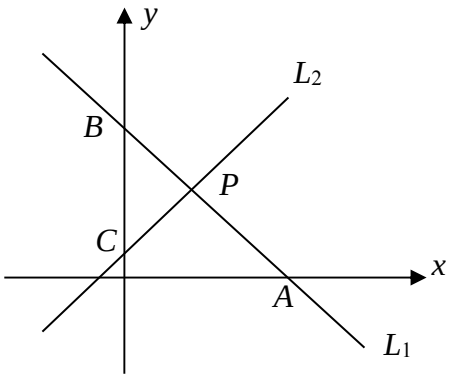
15. If the straight line L passes through (4, −5) and has slope $\frac{-1}{2}$, find

(a) the equation of L; (2 marks)
(b) the x-intercept and the y-intercept of L. (2 marks)

16. Express $\frac{-i}{3-i}$ in the form of $a+bi$, where a and b are real numbers. (3 marks)

17. It is given that the equation $2x(1-2kx)=3x+2$ has real roots and $k \neq 0$. Find the range of k. (3 marks)

18. In the figure, the two straight lines $L_1 : 3x + 2y - 12 = 0$ and $L_2 : x - y + 1 = 0$ intersect at P. L_1 cuts the x-axis and the y-axis at A and B respectively and L_2 cuts the y-axis at C.



(a) Find the coordinates of P. (2 marks)

(b) Someone claims that the area of ΔPBC is smaller than that of ΔPCA. Is the claim correct? Explain your answer. (4 marks)

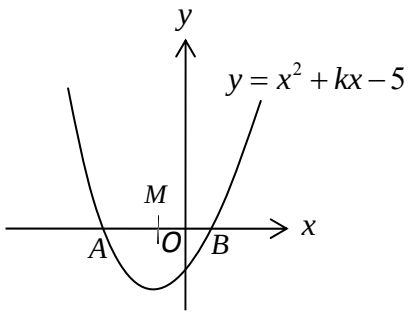
19. If α and β are the roots of the equation $x^2+9x+7=0$, find the value of the following expressions.

(a) $\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)$, (4 marks)

(b) $\alpha^2-9\beta+2$. (3 marks)

20. Simplify $\frac{\sin(180°-\theta)}{\cos(360°+\theta)\tan(270°+\theta)}$. (3 marks)

21. In the figure, the graph of $y = x^2 + kx - 5$ cuts the x-axis at two points A and B. If the distance between A and B is 6 units and $k > 0$,



(a) find the value of k; (5 marks)

(b) find the coordinates of the mid-point M of A and B. (2 marks)

22. It is given that $f(x) = ax^2 + 24x + 36$ and $f(-1) = 15$.

(a) Find $f(x)$. (2 marks)

(b) The graph of $y = f(x)$ cuts the x-axis at $A(\alpha, 0)$ and $B(\beta, 0)$, where $\alpha < \beta$. Find the values of α and β. (3 marks)

(c) The graph of $y = f(x)$ cuts the y-axis at C and the vertex of the graph $y = f(x)$ is denoted as Q.

(i) Find the area of ΔABQ. (3 marks)

(ii) Find the equation of the straight line that passes through the circumcenters of ΔABC and ΔABQ. (2 marks)

23. (a) Rewrite $3\cos^2 x + 9\sin x - 11$ in the form $A(\sin x - B)^2 + C$, where A, B and C are constants. (5 marks)

(b) Write down the maximum value and minimum value of $3\cos^2 x + 9\sin x - 11$, where $60° \le x \le 270°$. (2 marks)

******* END OF PAPER *******