

TB(3B) Ch.7 Areas and Volumes (III) Conventional Questions

1. [16-17 Standardized Test #4]

In **Figure 3(a)**, solid X is formed by fixing a right circular cone of base radius 3 cm and height 8 cm onto a hemisphere with the same radius.

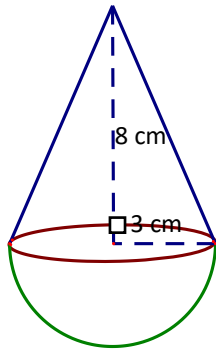


Figure 3(a): Solid X

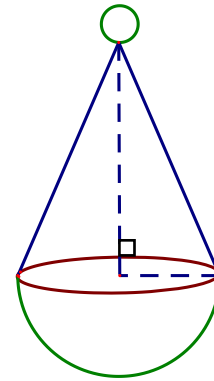


Figure 3(b): Solid X'

- (a) Find the volume of solid X . (3 marks)
- (b) Another solid Y is similar to X such that the total surface area of solid X : the total surface area of solid $Y = 9 : 16$. Find the volume of solid Y . (2 marks)
- (c) In **Figure 3(b)**, solid X' is formed by fixing a sphere of diameter 2 cm on the tip of solid X . Another solid Y' is formed by fixing a sphere of diameter 3 cm on the tip of solid Y . Are solid X' and solid Y' similar? Explain your answer. (2 marks)

2. [16-17 Mid-year Exam #11]

Figure 4(a) shows a rectangular pyramid $VABCD$. It is made of sand and V is vertically above D . VAB , VBC , VCD and VDA are right-angled triangles. It is given that $AB = 12$ cm, $BC = 8$ cm and $VD = 16$ cm.

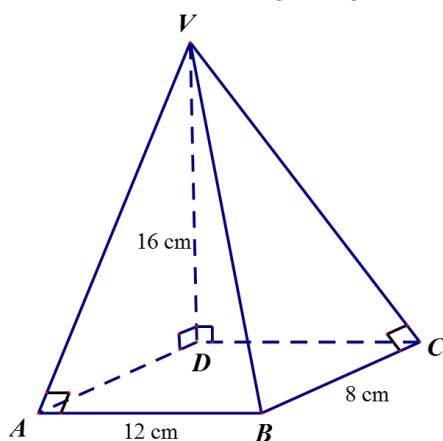


Figure 4(a)

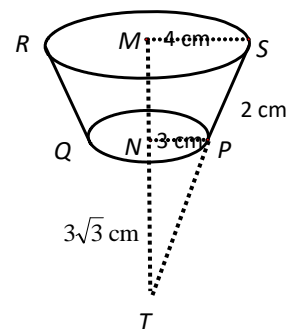


Figure 4(b)

- (a) (i) Find VA and VC . (1 mark)
 (ii) Find the total surface area of the pyramid. (2 marks)
- (b) **Figure 4(b)** shows a cup $PQRS$ in a shape of a conical frustum with lower base radius NP 3 cm and

upper base radius MS 4 cm. Its slant height PS is 2 cm and NT is $3\sqrt{3}$ cm. Show that the capacity of the cup is $\frac{37\sqrt{3}}{3}\pi$ cm³. (2 marks)

3. [16-17 Final Exam #17]

Figure 7 shows a right circular conical container, which holds some water in it. It is known that the depth of the water is 6 cm and the radius of the circular surface of the water is 3 cm.

(a) Find the wet surface area of the container.

(2 marks)

(b) If a marble is totally submerged in the water and the wet surface area of the container increases by 56.25%, find the volume of the marble.

(2 marks)

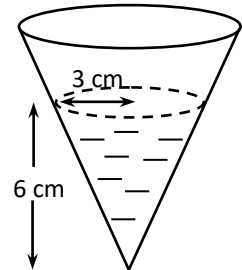


Figure 7

4. [17-18 Mid-year Exam #8]

Figure 4 shows a pyramid $VABCD$ with a rectangular base. It is given that $AB = 12$ cm, $BC = 9$ cm and the height of the pyramid is 5 cm.

(a) Find the volume of $VABCD$.

(2 marks)

(b) If AB is increased by 20%, BC remains unchanged, and the height of the pyramid is reduced by 30%, find the percentage decrease in its volume.

(3 marks)

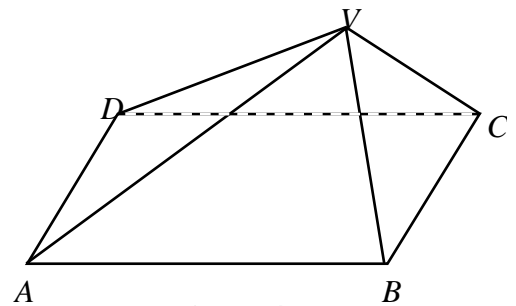


Figure 4

5. [17-18 Mid-year Exam #9]

In Figure 5, it shows a right pyramid A with height VO . Its base $PQRS$ is a square with sides 4 cm. The volume of the pyramid is $\frac{32\sqrt{7}}{3}$ cm³. It is given that M and N are the mid-points of PQ and QR respectively.

(a) Find the height VO . Keep your answer in surd form.

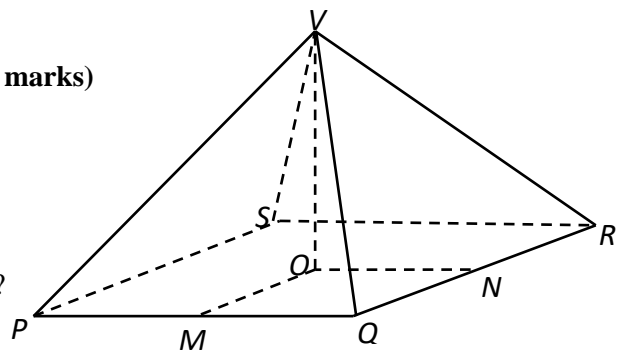
(2 marks)

(b) Find the length of the slant edge of the pyramid.

(3 marks)

(c) There is another right pyramid B with slant edge 20% longer than that of right pyramid A , and its base is a square with sides 4 cm. Kelly claimed that the area of lateral faces of right pyramid B is larger than that of right pyramid A by more than 22.5%. Do you agree with her claim? Explain your answer.

(3 marks)



Figure

6. [17-18 Standardized Test #6]

Figure 2a shows a solid metal cuboid. It is melted and recast into two similar solid right pyramids with square bases as shown in Figure 2b. The ratio of the base area of the smaller pyramid $ABCDE$ to the base area of the larger pyramid $FGHIJ$ is 4 : 9.

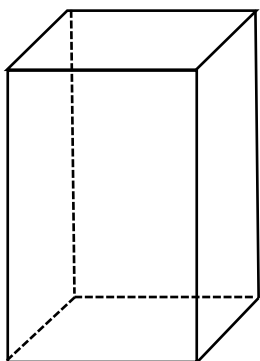


Figure 2a

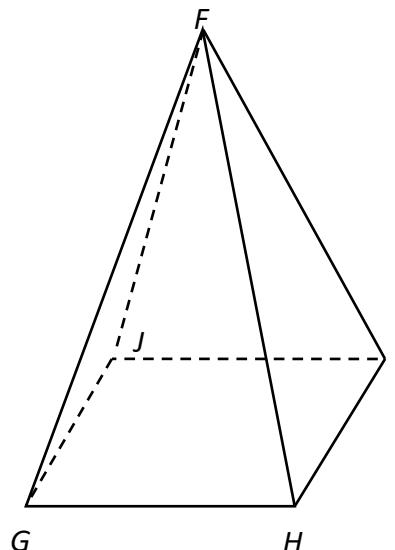
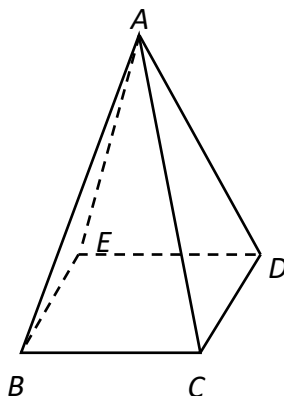


Figure 2b

- (a) (i) Write down the ratio of $BC : GH$. (1 mark)
 (ii) It is given that the volume of the larger pyramid $FGHIJ$ is $V \text{ cm}^3$. Express the volume of the smaller pyramid in terms of V . (1 mark)
- (b) (i) It is given that the base area and the height of the cuboid is 42 cm^2 and 15 cm respectively. Find the volume V of the larger pyramid. (2 marks)
 (ii) If the height of the larger pyramid is 18 cm , find the side length of the base of the larger pyramid. (2 marks)
 (iii) Find the total surface area of the smaller pyramid. (2 marks)

7. [17-18 Final Exam #9]

It is given that $h \text{ cm}$ and $H \text{ cm}$ are the original height and the new height of a cylinder. If the original radius $r \text{ cm}$ of the cylinder is increased by 25%, find the percentage change in the height of the cylinder if its volume remains unchanged. (3 marks)

8. [17-18 Final Exam #14]

- (a) An inverted right circular conical vessel of base radius 14 cm and height 41 cm is fully filled with water. Find the volume of water in the vessel in terms of π . (2 marks)
- (b) **Figure 5** shows a frustum container of upper base radius 15 cm, lower base radius 13 cm and height 14 cm. Find the volume of the container. (2 marks)
- (c) The water in the vessel in (a) is now poured into the container. If 6 solid metal spheres of radius r cm is then put into the container and the spheres are totally immersed in the water. By setting an inequality, find the maximum length of radius r cm, correct to 3 significant figures, such that the water will not overflow. (3 marks)

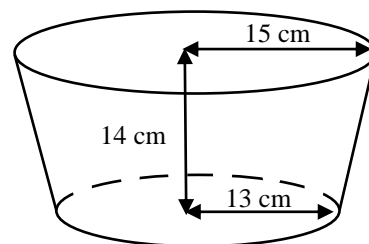


Figure 5

9. [18-19 Mid-year Exam #9]

Figure 4 shows a right pyramid $VABCD$ with height 15 cm and a square base of side y cm. Its volume is 1280 cm^3 .

- (a) (i) Find the value of y . (2 marks)
 (ii) Find the height of $\triangle VAB$ with reference to the base AB . (1 mark)
- (b) Find the total surface area of the pyramid. (2 marks)

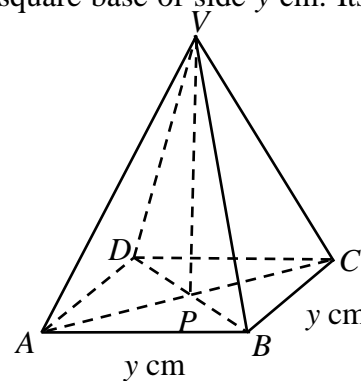


Figure 4

10. [18-19 S Test 2 #3]

In **Figure 3**, $VABCD$ is a right pyramid with a rectangular base. VE is the height of the pyramid.

- (i) Name the projection of line VA on plane $ABCD$. (1 mark)
 (ii) Name the angle between line VA and plane $ABCD$. (1 mark)
- (a) If M is a point on BC and $\angle VME$ is the angle between planes VBC and $ABCD$, what is the geometric relationship between M and BC ? (1 mark)

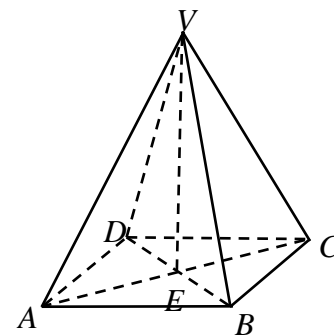


Figure 3

11. [18-19 S Test 2 #5]

In **Figure 4**, water of volume $9\pi \text{ cm}^3$ is poured into an inverted right circular conical vessel. If the depth of water is the same as the radius of the water surface, find

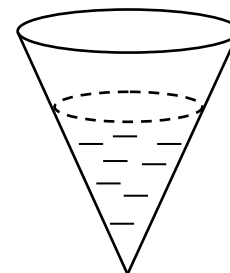


Figure 4

- (a) (i) the depth of water, and
 (ii) the wet surface area of the vessel. (4 marks)
- (b) Amy claims that if an extra $54\pi \text{ cm}^3$ of water is poured into the vessel and the water does not overflow, the percentage increase in the wet surface area of the vessel is less than 270%. Do you agree? Explain your answer. (2 marks)

12. [18-19 Final Exam #8]

A sector of area $500\pi \text{ cm}^2$ and radius 25 cm, as shown in **Figure 2(a)**, is folded to form a right circular cone as shown in **Figure 2(b)**.

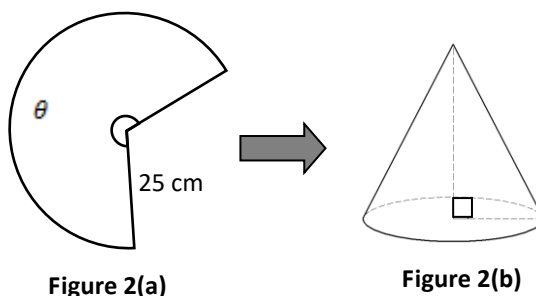


Figure 2(a)

Figure 2(b)

- (a) Find θ . (1 mark)
- (b) (i) Find the base radius of the cone in **Figure 2(b)**. (2 marks)
 (ii) Find the volume of the cone in **Figure 2(b)**. (2 marks)

13. [20-21 Standardized Test #2]

Figure 2 shows a right pyramid with a square base. The length of the square base is 20 cm and the volume of the pyramid is 3200 cm^3 .

- (a) Find the height of the pyramid. (2 marks)
- (b) Sketch the net of the pyramid in the box provided. (1 mark)
- (c) Find the total surface area of the pyramid. (2 marks)

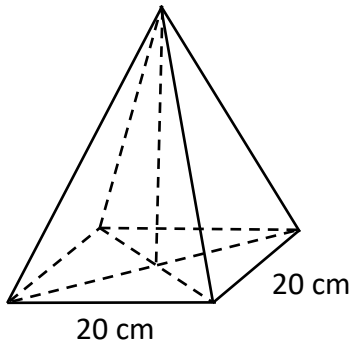


Figure 2



14. [20-21 Standardized Test #4]

Figure 4 shows a container in the shape of a hollow sphere with uniform thickness 1 cm. It is given that the capacity of the container is 4000 cm^3 .

- (a) Find the inner radius of the container. (2 marks)
- (b) Find the outer surface area of the container. (2 marks)

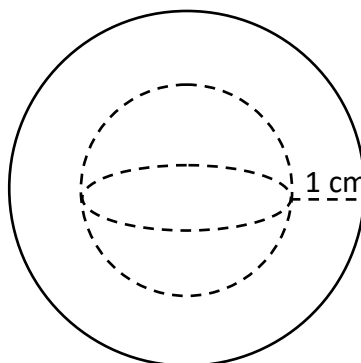


Figure 4

15. [20-21 Standardized Test #6]

Figure 5 shows a frustum $ABCDEFGH$. The lower base is a square $ABCD$ of side 30 cm and the upper base is a square $EFGH$ of side 20 cm. The volume of the frustum is 3800 cm^3 . If the height of the frustum is h cm, find h . (3 marks)

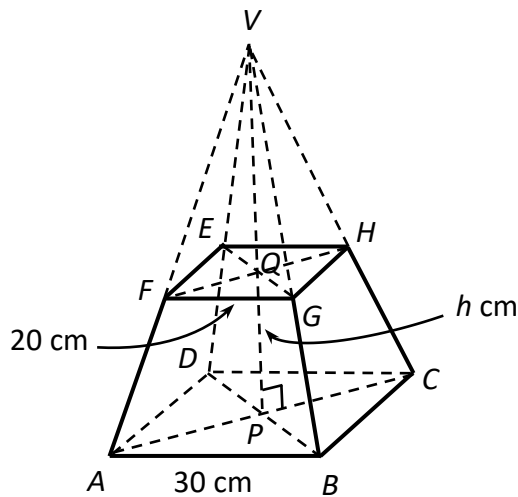


Figure 5

16. [20-21 Final Exam, #10]

Figure 4 shows a paper cup in the shape of an inverted right circular cone with base diameter of 18 cm. The capacity of the cup is $1\,080\pi\text{ cm}^3$.

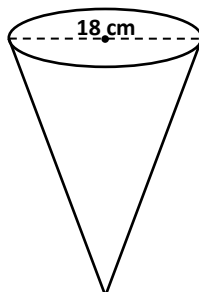


Figure 4

- (a) Find the slant height of the cup. (3 marks)
- (b) Emily claims that if the cup is cut along a slant height to form a sector, the angle of the sector is an acute angle. Do you agree? Explain your answer. (2 marks)

17. [20-21 Final Exam, #14]

In Figure 7, a solid metallic right pyramid S with square base is divided into a right pyramid P and a frustum Q . The heights of P and Q are 12 cm and 4 cm respectively.

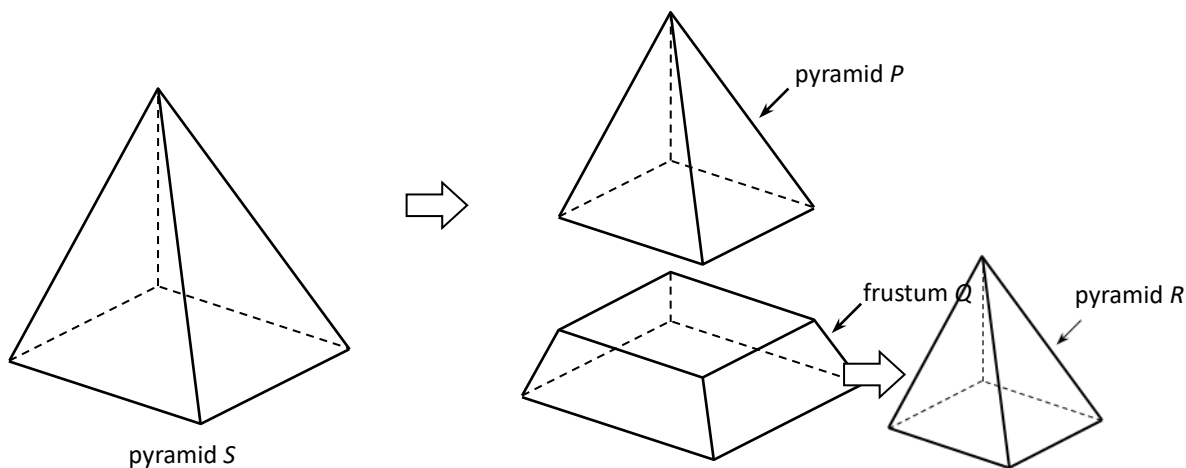


Figure 7

- (a) Find the ratio of the volume of P to the volume of Q . (3 marks)
- (b) Suppose the volume of P is 324 cm^3 . If frustum Q is melted and recast into a right pyramid R with square base of base area 222 cm^2 , are P and R similar? Explain your answer. (3 marks)