# Trigonometric Relations
# Conventional Questions

**1. [14-15 Standardized Test, #2]**

Without using calculator, find the acute angle $\theta$ in $\sin\theta = \dfrac{2\cos^2 30°}{\tan 60°}$. **(2 marks)**

**2. [14-15 Standardized Test, #3]**

It is given that sin $\theta$ = 0.6, where $\theta$ is an acute angle. Without finding the value of $\theta$, find the value of $\dfrac{\sin^2\theta}{\cos\theta - \tan\theta}$. **(3 marks)**

**3. [14-15 Standardized Test, #6]**

**(a)** Prove that $(\cos\alpha\cos\beta - \sin\alpha\sin\beta)(\cos\alpha\cos\beta + \sin\alpha\sin\beta) \equiv 1 - \sin^2\alpha - \sin^2\beta$. **(3 marks)**

**(b)** Hence, find the value of

$$\frac{(\cos\alpha\cos\beta - \sin\alpha\sin\beta)(\cos\alpha\cos\beta + \sin\alpha\sin\beta) - \cos^2\alpha}{\tan^2\beta} - \sin^2\beta.$$ **(2 marks)**

**4. [14-15 Final Exam, #1]**

**(a)** Prove the identity $\dfrac{\tan\theta(1-\sin^2\theta)}{\cos\theta} \equiv \cos(90° - \theta)$. **(3 marks)**

**(b)** It is given that $\tan\theta = \dfrac{3}{4}$. Without finding the value of $\theta$, find the values of $\sin\theta$ and $\cos\theta$. **(2 marks)**

**5. [15-16 Standardized Test, #2]**

It is given that $\sin\theta = \dfrac{\sqrt{2}}{3}$, where $\theta$ is an acute angle. Find, without finding the value of $\theta$, the values of $\tan\theta$ and $\cos\theta$. Leave your answers in surd form. **(2 marks)**

**6. [15-16 Standardized Test, #5]**

**(a)** Prove that $\dfrac{\sqrt{\sin^2\theta - \cos^4(90° - \theta)}}{\tan\theta} \equiv \cos^2\theta$, where $\theta$ is an acute angle. **(2 marks)**

**(b)** Hence, or otherwise, solve $\dfrac{\sqrt{\sin^2 30° - \cos^4 60°}}{\tan 30°} = \tan(x + 20°) - \dfrac{1}{4}$, where $x$ is an acute angle. **(2 marks)**

**7. [15-16 Final Exam, #12]**

**(a)** It is given that $\tan\theta = \frac{3}{4}$, where $\theta$ is an acute angle. Find the value of $\frac{1-\cos\theta}{\sin\theta}$ without evaluating $\theta$. **(2 marks)**

**(b)** Simplify $\sin\phi + (\cos\phi - 1)\tan(90° - \phi)$. **(3 marks)**

**8. [15-16 Final Exam, #18]**

**(a)** Prove that $\frac{1-\cos\theta}{\sin\theta} \equiv \frac{\sin\theta}{1+\cos\theta}$. **(2 marks)**

**(b)** In **Figure 10**, *BCD* is a straight line and *CB* = *CA* = 1, $\angle ABD = \frac{\phi}{2}$ and $\angle ACD = \phi$. Express $\tan\frac{\phi}{2}$ in terms of $\phi$. **(2 marks)**

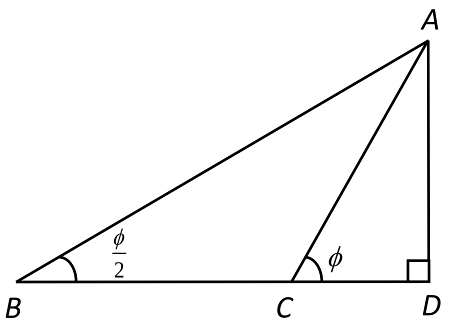


**Figure 10**

**9. [16-17 Standardized Test, #2]**

It is given that $\cos\theta = \frac{\sqrt{5}}{4}$, where $\theta$ is an acute angle. Without finding the value of $\theta$, write down the values of $\sin\theta$ and $\tan\theta$ in surd form. **(2 marks)**

**10. [16-17 Final Exam, #8]**

Evaluate $\frac{\sin 60°}{\cos 60° \tan^2 60°}$. Leave the answer in surd form. **(2 marks)**

**11. [16-17 Final Exam, #10]**

If $\tan(90° - \theta) = 3$, without using a calculator, find the value of $\frac{\cos^3\theta + \cos\theta\sin^2\theta}{\sin\theta}$.

**(3 marks)**

**12. [16-17 Final Exam, #19]**

Find the value of

$\sin 1°\cos 1°\tan 1° - \sin 2°\cos 2°\tan 2° + \sin 3°\cos 3°\tan 3° - \ldots + \sin 89°\cos 89°\tan 89°$. **(2 marks)**

**13. [17-18 Standardized Test, #3]**

It is given that $\tan\theta = \frac{\sqrt{5}}{2}$, where $\theta$ is an acute angle. Find the value of $\sin\theta\cos\theta$ in surd form without evaluating $\theta$. **(2 marks)**

**14. [17-18 Final Exam, #4]**

Evaluate $\frac{\tan 60°\cos^2 45°}{\sin 30°}$ without using calculator. Leave the answer in surd form. **(2 marks)**

**15. [17-18 Final Exam, #5]**

$\Delta ABC$ is a right-angled triangle where $\angle A = \theta$ and $\angle C = 90°$. If $AB = 7$ and $BC = 2$, find the value of $\tan\theta$ in surd form without evaluating $\theta$. **(2 marks)**

**16. [17-18 Final Exam, #10]**

**(a)** Prove that $\sin\theta\tan^2(90° - \theta) \equiv \frac{1}{\sin\theta} - \sin\theta$. **(2 marks)**

**(b)** Hence, or otherwise, solve $\sin\theta\tan^2(90° - \theta) - \frac{1}{\sin\theta} = -\frac{1}{2\tan(90° - \theta)}$, where $0° < \theta < 90°$. **(2 marks)**

**17. [18-19 Standardized Test 2, 4]**

Evaluate $\frac{\cos 30°\sin 45°\tan 60°}{\sin 60°}$ without using calculator. Leave your answer in surd form.

**(2 marks)**

**18. [18-19 Standardized Test 2, 7]**

**(a)** Prove that $\cos^2\theta - \cos^2\theta\sin^2\theta \equiv \sin^2\theta\cos^2\theta\tan^2(90° - \theta)$, where $\theta$ is an acute angle.

**(2 marks)**

**(b)** It is given that $\sin^4\theta - \sin^4(90° - \theta) = 1 - k^2$. Show that $k = \sqrt{2}\cos\theta$. **(3 marks)**

**19. [18-19 Final Exam, #7]**

Without using a calculator,

**(a)** solve $\sin(60° - 2\theta) = \cos 50°$, **(2 marks)**

**(b)** find the value of $\tan^2 60° - \frac{1}{4\cos 60°} - \sin^2 45°$. **(2 marks)**

**20. [18-19 Final Exam, #16]**

If $a + b = 90°$, simplify $(\sin a + \sin b)^2 - 2\cos a\cos b$. **(2 marks)**