Trigonometric Relations
Conventional Questions

1. [13-14 Standardized Test 2, #5]

In **Figure 5**, *ABC* // *DEF*, $BE \perp AC$, *BC* = *BE*, *CE* = $2\sqrt{2}$ cm and $\angle AED = 30^\circ$. Find *AE*. **(4 marks)**

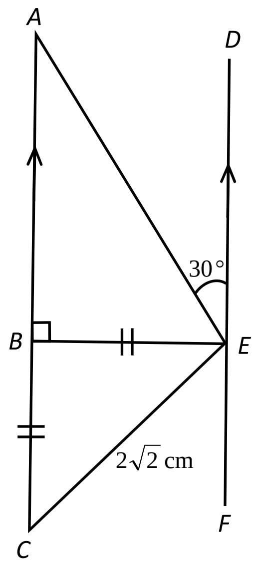

Figure 5

2. [13-14 Final Exam, #12]

It is given that $\sin(90^\circ - \theta) = \frac{1}{5}$ where θ is an acute angle. Without using a calculator, find

(a) the values of $\cos\theta$, $\sin\theta$ and $\tan\theta$ in surd forms, **(3 marks)**

(b) the value of ϕ in $\frac{24\cos(\phi + 20^\circ)\tan(\phi + 20^\circ)}{25\cos^2(90^\circ - \theta)} = \frac{\cos(2\phi + 40^\circ)}{24\tan^2(90^\circ - \theta)}$ where ϕ is an acute angle, by using the result of part **(a)**. **(4 marks)**

3. [14-15 Standardized Test, #2]

Without using calculator, find the acute angle θ in $\sin\theta = \frac{2\cos^2 30^\circ}{\tan 60^\circ}$. **(2 marks)**

4. [14-15 Standardized Test, #3]

It is given that sin θ = 0.6, where θ is an acute angle. Without finding the value of θ, find the value of $\frac{\sin^2\theta}{\cos\theta - \tan\theta}$. **(3 marks)**

5. [14-15 Standardized Test, #6]

(a) Prove that $(\cos\alpha\cos\beta - \sin\alpha\sin\beta)(\cos\alpha\cos\beta + \sin\alpha\sin\beta) \equiv 1 - \sin^2\alpha - \sin^2\beta$. **(3 marks)**

(b) Hence, find the value of

$$\frac{(\cos\alpha\cos\beta - \sin\alpha\sin\beta)(\cos\alpha\cos\beta + \sin\alpha\sin\beta) - \cos^2\alpha}{\tan^2\beta} - \sin^2\beta.$$ **(2 marks)**

6. [14-15 Final Exam, #1]

(a) Prove the identity $\dfrac{\tan\theta(1-\sin^2\theta)}{\cos\theta} \equiv \cos(90° - \theta)$. **(3 marks)**

(b) It is given that $\tan\theta = \dfrac{3}{4}$. Without finding the value of θ, find the values of $\sin\theta$ and $\cos\theta$. **(2 marks)**

7. [15-16 Standardized Test, #2]

It is given that $\sin\theta = \dfrac{\sqrt{2}}{3}$, where θ is an acute angle. Find, without finding the value of θ, the values of $\tan\theta$ and $\cos\theta$. Leave your answers in surd form. **(2 marks)**

8. [15-16 Standardized Test, #5]

(a) Prove that $\dfrac{\sqrt{\sin^2\theta - \cos^4(90° - \theta)}}{\tan\theta} \equiv \cos^2\theta$, where θ is an acute angle. **(2 marks)**

(b) Hence, or otherwise, solve $\dfrac{\sqrt{\sin^2 30° - \cos^4 60°}}{\tan 30°} = \tan(x + 20°) - \dfrac{1}{4}$, where x is an acute angle. **(2 marks)**

9. [15-16 Final Exam, #12]

(a) It is given that $\tan\theta = \dfrac{3}{4}$, where θ is an acute angle. Find the value of $\dfrac{1-\cos\theta}{\sin\theta}$ without evaluating θ. **(2 marks)**

(b) Simplify $\sin\phi + (\cos\phi - 1)\tan(90° - \phi)$. **(3 marks)**

10. [15-16 Final Exam, #18]

(a) Prove that $\dfrac{1-\cos\theta}{\sin\theta} \equiv \dfrac{\sin\theta}{1+\cos\theta}$. **(2 marks)**

(b) In **Figure 10**, BCD is a straight line and $CB = CA = 1$, $\angle ABD = \dfrac{\phi}{2}$ and $\angle ACD = \phi$. Express $\tan\dfrac{\phi}{2}$ in terms of ϕ. **(2 marks)**

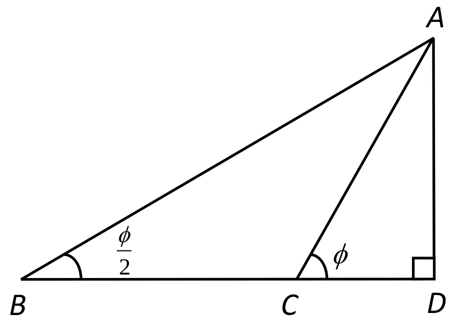

Figure 10

11. [16-17 Standardized Test, #2]

It is given that $\cos\theta = \dfrac{\sqrt{5}}{4}$, where θ is an acute angle. Without finding the value of θ, write down the values of $\sin\theta$ and $\tan\theta$ in surd form. **(2 marks)**

12. [16-17 Final Exam, #8]

Evaluate $\dfrac{\sin 60°}{\cos 60° \tan^2 60°}$. Leave the answer in surd form. **(2 marks)**

13. [16-17 Final Exam, #10]

If $\tan(90° - \theta) = 3$, without using a calculator, find the value of $\dfrac{\cos^3\theta + \cos\theta\sin^2\theta}{\sin\theta}$.

(3 marks)

14. [16-17 Final Exam, #19]

Find the value of

$\sin 1° \cos 1° \tan 1° - \sin 2° \cos 2° \tan 2° + \sin 3° \cos 3° \tan 3° - \ldots + \sin 89° \cos 89° \tan 89°$.

(2 marks)

15. [17-18 Standardized Test, #3]

It is given that $\tan\theta = \dfrac{\sqrt{5}}{2}$, where θ is an acute angle. Find the value of $\sin\theta\cos\theta$ in surd form without evaluating θ. **(2 marks)**

16. [17-18 Final Exam, #4]

Evaluate $\dfrac{\tan 60° \cos^2 45°}{\sin 30°}$ without using calculator. Leave the answer in surd form. **(2 marks)**

17. [17-18 Final Exam, #5]

ΔABC is a right-angled triangle where $\angle A = \theta$ and $\angle C = 90°$. If $AB = 7$ and $BC = 2$, find the value of $\tan\theta$ in surd form without evaluating θ. **(2 marks)**

18. [17-18 Final Exam, #10]

(a) Prove that $\sin\theta\tan^2(90° - \theta) \equiv \dfrac{1}{\sin\theta} - \sin\theta$. **(2 marks)**

(b) Hence, or otherwise, solve $\sin\theta\tan^2(90° - \theta) - \dfrac{1}{\sin\theta} = -\dfrac{1}{2\tan(90° - \theta)}$,

where $0° < \theta < 90°$. **(2 marks)**