ST. STEPHEN'S GIRLS' COLLEGE
Mid-Year Examination 2017 – 2018

Form 3   **MWC, WYL, SCHL**
157 students

Mathematics
Time allowed: 1 hour 30 minutes
Question/Answer Paper

Please read the following <u>instructions</u> very carefully.

Class	
Class No.	
Name	

1. This paper consists of TWO sections, A and B.

2. Write your class, class number and name in the spaces provided on this cover.

3. This paper carries 100 marks.   Attempt ALL questions in this paper.   Write your answers in the spaces provided in this Question/Answer Paper.

4. The diagrams in this paper are not necessarily drawn to scale.

5. Unless otherwise specified, numerical answers should either be exact or correct to 3 significant figures.

For Markers' Use Only		
1 – 15		(40)
16 – 17	(3)	(4)
18 – 19	(4)	(7)
20 – 21	(6)	(6)
22 – 23	(5)	(6)
24 – 25	(4)	(6)
26 – 27	(3)	(6)
TOTAL		(100)

Section A (40%)

All rough work should be done on the rough work paper provided, but will not be marked.

	Answer	**Marks**
1. Factorize the following polynomials.	1.	
(a) $4m^2 - 25n^2$	(a) _____	2
(b) $x^2 + 5x - 14$	(b) _____	2
(c) $2x^2 - xy - 10y^2$	(c) _____	2
2. Express the following numbers in scientific notation.	2.	
(a) 0.000 0034	(a) _____	1
(b) −270 000 000	(b) _____	1
3. Evaluate $\left(\dfrac{1}{4}\right)^{500}(2^{500})^2$.	3. _____	2
4. Convert the decimal number $11\times16^7 + 495$ into a hexadecimal number.	4. _____	2
5. A sum of $20 000 is deposited at an interest rate of 12% p.a. for 3 years compounded quarterly. Find the amount received correct to the nearest dollar.	5. _____	2
6. The value of a mobile phone decreases by 8% every 3 months. Mary bought the mobile phone one year ago at $5 400. Find its present value, correct to 1 decimal place.	6. _____	2
7. This month, Mr. Chan's family consumes 14% less electricity than that last month, but the charge per unit of electricity is increased by 5% as compared to last month. Find the percentage change in the charge of electricity over these two months.	7. _____	2
8. Determine whether each of the following statements must be true.	8.	
(a) If $x < y$, then $3x - 2 > 3y - 2$.	(a) True / False	1
(b) If $a > b > 0$, then $\dfrac{-2}{a} > \dfrac{-2}{b}$.	(b) True / False	1
(c) If $p < q$, then $p^2 < q^2$.	(c) True / False	1
	Subtotal:	/21

9. In a bakery, the prices of a cream cake and a cheese tart are $15 and $18.5 respectively. Miss Lee has to buy a total of 15 pieces of cakes and tarts from the bakery.   If she has $250 and she wants to buy as many cheese tarts as possible, how many cream cakes should be bought?

9. _____ 3

10. The following table shows the number of times that S3 students of a school watched movies last month.

| Number of times watching movies	0	1	2	3	4	5	6
Frequency	28	41	31	32	24	21	23

Find the mean, median and mode of the number of times that S3 students of a school watched movies last month.

10.
Mean   =_____ 1
Median =_____ 1
Mode   =_____ 1

11. The median of the five numbers 20, $x + 1$, $x + 5$, $x – 4$ and $x + 19$ is 15. Find the mean of these five numbers.

11. _____ 2

12. The following cumulative frequency polygon shows the number of books S2 students in a school read last year.

Number of books S2 students in a school read last year

12.
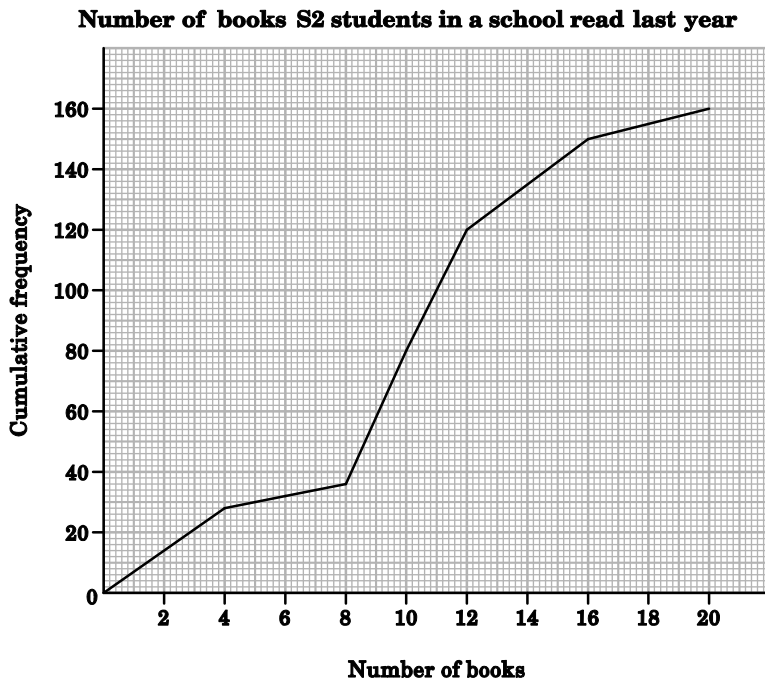
(a) Find the 80th percentile.

(a) _____ 2

(b) Students who read 12 books or more were given certificates last year. Find the percentage of students who got the certificate.

(b) _____ 2

Subtotal: /12

13. In the figure, O is the orthocentre of $\triangle ABC$, where $\angle OAC = 30°$ and $\angle OCA = 20°$. Find $\angle ABC$.

13. _____   2
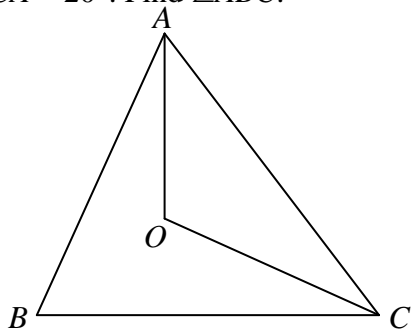
14. The perimeter of sector AOB is 10 cm and $\angle AOB = 30°$, where O is the centre. Find the radius of sector AOB, correct to 3 significant figures.

14. _____   2

15. In the figure, $OQRP$ is a sector with radius 6 cm. If $\angle POQ = 120°$ and OR is the angle bisector of $\angle POQ$, find the area of the shaded region, correct to 3 significant figures.
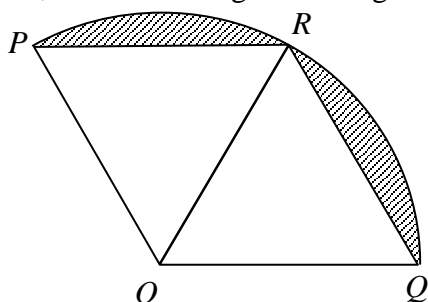
15. _____   3

Subtotal: /7

Section B (60%)

All working must be clearly shown in the spaces provided.

16. Simplify $(-4p^{-3}q^3)^{-2}\left(\dfrac{2p^2}{q}\right)^4$ and express the answer with positive indices.   (3 marks)

_____

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17. (a) Solve the inequality $x - \dfrac{3x+4}{5} \geq \dfrac{1-x}{3} - 6$.

    (b) If x is an integer, find the least possible value of x.   (4 marks)

18. Given that $2x < \dfrac{3x}{2} - \dfrac{1}{4}$ and $y = \dfrac{3}{4}(2-x)$, find the range of values of y.   (4 marks)

19. The government of city A imports water from cities X and Y.   City X agrees to supply a total amount of 1.52×10^{12} L of water, while the water supply from city Y is 9.5×10^{11} L more than that from city X.   It is known that the population of city A is 7.5×10^{6} and the average yearly water consumption of city A is 4.56×10^{4} L per person.

    (a)   Find the total amount of water supply from city X and city Y.

    (b)   Find the water consumption of city A in a year.

    (c)   The government of city A claims that if the yearly water consumption of city A remains unchanged, the total water supply from cities X and Y is enough for 11 years of consumption for the people in city A.   Do you agree? Explain your answer.   (7 marks)

20. The following table shows the test marks of Peter and Tom in 4 subjects.

	Chinese	English	Mathematics	Science
Peter	62	70	60	46
Tom	55	58	x	60
Weight	2	3	2	1

(a) Find the weighted mean mark of Peter. (1 mark)

(b) If Peter and Tom have the same weighted mean mark, find the value of x. (2 marks)

(c) Suppose the weights of Mathematics and Science are doubled. Find the minimum value of x (if x is an integer) so that Tom has a better performance than Peter. (3 marks)
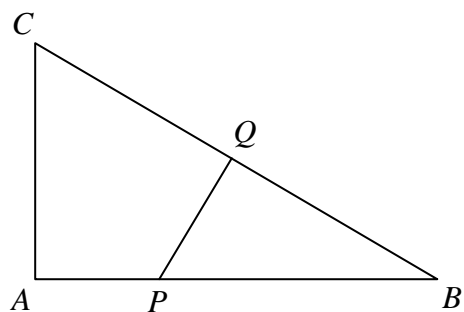
21. In the figure, PQ is the perpendicular bisector of BC and $\angle BAC = 90°$.

    (a) Prove that $\Delta ABC \sim \Delta QBP$.   (3 marks)

    (b) If $PQ = 3$ and $BQ = 4$, hence or otherwise, find the perimeter of ΔABC.   (3 marks)
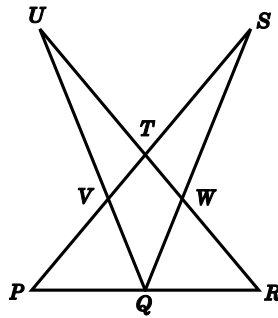
22. In the figure, *PQR*, *UTWR*, *STVP*, *UVQ* and *SWQ* are straight lines. *UQ* and *SQ* are the angle
    bisectors of $\angle PQS$ and $\angle UQR$ respectively. Given that $PQ = QR$, $\angle QUR = 20°$ and $\angle SPR = 40°$.
    (a) Find $\angle UQS$.   (2 marks)
    (b) Prove that $\Delta UQR \cong \Delta SQP$.   (3 marks)

U   S

T

V W

P   Q   R

23. A cylindrical metal pillar, with base radius 4 cm, stands upright on the base of the cylindrical vessel, with base diameter 20 cm, and the pillar is taller than the vessel, as shown in the figure. The original depth of water is h cm. If the cylindrical metal pillar is removed from the vessel, the depth of water is 12.6 cm.
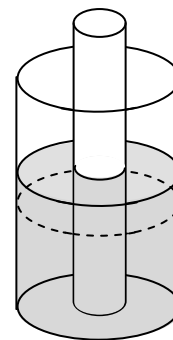
    (a) Find the value of h. (3 marks)

    (b) Find the total area of the wet surfaces of the pillar (including the base) before removing it from the vessel. Give your answer in terms of π. (3 marks)

24. The volumes of carrot juice, apple juice and water in bottle A are in the ratio 3 : 2 : 4, while those in bottle B are in the ratio 1 : 4 : 1. 800 cm^3 of the liquid in bottle A is mixed with the liquid in bottle B. If the volume of carrot juice in the mixture is less than the volume of apple juice in the mixture by at least 30% of the total volume of the mixture, find the minimum possible volume of the liquid from bottle B, correct to the nearest cm^3. (4 marks)

25. Daniel borrowed a loan of $150 000 from a bank. Interest was calculated at the end of every year at an interest rate of 12% per annum. He planned to repay x at the end of every year. It was assumed that the interest was calculated before Daniel made the repayment.

   (a) Find, in terms of x, the amount he still owed the bank after the ***first*** repayment.
      (*Express the answer as an expanded and simplified algebraic expression.*) (1 mark)

   (b) It is given that the amount Daniel still owed the bank after the second repayment is $18 560.
      (i) Find x. (2 marks)
      (ii) Daniel claimed that he should increase the repayment by $8 000 in order to fully repay his loan in two years. Do you agree? Explain your answer. (3 marks)

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26. Simplify $\dfrac{3^{2n+2}-9^{n-1}}{3^{2n}+3^{2n+1}}$, where *n* is an integer. (3 marks)

27. (a) Factorize $x^3 - 64$. (1 mark)

    (b) Factorize $x^2 + 25x - 116$. (1 mark)

    (c) Using (a) and (b), or otherwise, factorize $2x^3 - x^2 - 25x - 12$. (4 marks)

Simplify $\dfrac{3^{2n+2}-9^{n-1}}{3^{2n}+3^{2n+1}}$, where *n* is an integer.

End of Paper

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